1) Instances, Variants and Substitutions
   1. Suppose $\Theta_1$ and $\Theta_2$ are substitutions and there exist substitutions $\sigma_1$ and $\sigma_2$, such that $\Theta_1 = \Theta_2 \sigma_1$ and $\Theta_2 = \Theta_1 \sigma_2$. Show that there exists a variable-pure substitution $\gamma$, such that $\Theta_1 = \Theta_2 \gamma$.

   2. Which of the following clauses are Instances or Variants of each other?
      1. $p(x, y, z) :- q(x, y), r(f(z))$
      2. $p(x, b, f(z)) :- q(x, b), r(f(f(z)))$
      3. $p(v, w, f(z)) :- q(v, b), r(f(f(z)))$
      4. $p(z, w, v) :- q(z, w), r(f(v))$
      5. $p(f(x), y, f(z)) :- q(f(x), y), r(f(f(z)))$
      6. $p(f(x), y, z) :- q(f(x), y), r(f(z))$

2) The following Lemma shows that we only need to deal with Herbrand interpretations in order to find a model for any logic program:

   Let $C$ be a set of clauses and $\Sigma$ be any signature containing all symbols used in $C$. The grounding of $C$ with respect to $\Sigma$, denoted $C^*$ is the set of all ground instances of the signature $\Sigma$ of clauses in $C$. Let $I$ be a Herbrand interpretation and $C$ be a set of clauses.

   Prove that $I \models C$ if and only if $I \models C^*$.

3) Program Completion
   1. Let the definition of a predicate symbol $p$ be
      
      $p(y) :- q(y), \neg r(a,y)$.
      $p(f(z)) :- \neg q(z)$.
      $p(b)$.

      Give a completion of $p$.

   2. Let $P$ be a normal program and $\text{comp}(P)$ it's completion. Prove that $P$ is a logical consequence of $\text{comp}(P)$. Hint: $P$ is a logical consequence of $\text{comp}(P)$ if $I \models \text{comp}(P) \rightarrow I \models P$