Stratified Programs
Observation:
Every normal program is consistent (has a model), but this is not necessarily true for \( \text{comp}(P) \).

Example:

<table>
<thead>
<tr>
<th>Program P:</th>
<th>( \text{comp}(P) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \leftarrow \neg q )</td>
<td>( p \leftrightarrow \neg q )</td>
</tr>
<tr>
<td>( q \leftarrow \neg r )</td>
<td>( q \leftrightarrow \neg r )</td>
</tr>
<tr>
<td>( r \leftarrow \neg p )</td>
<td>( r \leftrightarrow \neg p )</td>
</tr>
<tr>
<td>( l = {p,q,r} ) is a model</td>
<td>By transitivity: ( p \leftrightarrow \neg p )</td>
</tr>
</tbody>
</table>

Thus there exists no model for \( \text{comp}(P) \)

Limit the use of negation in recursion
Definition:

A *level mapping* of a normal program is a mapping from its set of predicate symbols to the non-negative integers. We refer to the value of a predicate symbol under this mapping as the *level* of that predicate symbol.
Hierarchical Normal Program

Definition:
A normal program is hierarchical if it has a mapping such that in every program clause
A ← L₁, ..., Lₙ, the level of every predicate symbol occurring in the body is less than the level of A.

Observation:
not hierarchical:
relatedTo(x, y) ← relatedTo(y, x)
Stratification

Definition:
A normal program is stratified if it has a level mapping such that in every clause $A \leftarrow L_1, \ldots, L_n$,
- the level of the predicate symbol of every positive literal is less or equal to the level of $A$ and
- the level of each predicate symbol of every negative literal is less than the level of $A$. 
Example for Stratification

loves(x,y) ← friend(x,y)
loves(x,y) ← enemy(x,y)

enemy(x,y) ← ~friend(x,y)

friend(x,z) ← friend(x,y), friend(y,z)
friend(a,b) ←
friend(b,c) ←
Counterexample

\[\text{man}(x) \leftarrow \text{person}(x), \neg \text{woman}(x)\]

\[\text{woman}(x) \leftarrow \text{person}(x), \neg \text{man}(x)\]

\[\text{woman}(x) \leftarrow \text{person}(x), \neg \text{man}(x)\]

\[\text{man}(x) \leftarrow \text{person}(x), \neg \text{woman}(x)\]
Corollary:
Let $P$ be a stratified normal program. Then $\text{comp}(P)$ has a minimal normal Herbrand model. A normal Herbrand model assigns the equality relation to "=".
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Let P be a stratified normal program. then comp(P) has a minimal normal Herbrand model. A normal Herbrand model assigns the equality relation to „=“.
Local Stratification

Definition

A normal program is locally stratified if each atom in \( B_P \) can be assigned a countable ordinal level such that no atom

- positively depends of an atom of greater level
- negatively depends of an atom of equal or greater level.
Example for Local Stratification

\[ \text{even}(s(X)) \leftarrow \neg \text{even}(X). \]
\[ \text{even}(0). \]

\[ B_P: \]
\[ \{ \text{even}(0)^0, \text{even}(s(0))^1, \text{even}(s(s(0)))^2, \text{even}(\ldots)^3, \ldots \} \]
**Perfect Model**

**Definition**
Let $P$ be a normal program and $I$ a model. $I$ is a perfect model for a given level of $B_P$, if for every other model $J$, if a positive literal $p$ is the atom of least level in one model, but not in the other, then $p$ is in $J$.

In other words, atoms of higher level are preferred for the perfect model.

Przymusinski: All locally stratified programs have a perfect model, which is independent of the ranking system chosen.
Examples for Local Stratification

$\text{even}(s(X)) \leftarrow \neg \text{even}(X)$.
$\text{even}(0)$.
$\text{even}(0) \leftarrow q(X)$.

$B_P$:
$J=\{q(0)^0, \text{even}(0)^1, \text{even}(s(s(0)))^3, \ldots\}$
$I=\{\text{even}(0)^0, \text{even}(s(s(0)))^2, \ldots\}$