Models in First Order Logics

People and Knowledge Networks

Overview

First-order logic. Syntax and semantics.

Herbrand interpretations;

Clauses and goals;

Datalog.

First-order signature

First-order signature Σ consists of

- *con* — the set of constants of Σ;
- *fun* — the set of function symbols of Σ;
- *rel* — the set of relation symbols of Σ.

Terms

Term of Σ with variables in X:

1. Constant $c \in con$;
2. Variable $v \in X$;
3. If $f \in fun$ is a function symbol of arity n and $t_1, \ldots, t_n$ are terms, then $f(t_1, \ldots, t_n)$ is a term.

A term is ground if it has no variables

$var(t)$ — the set of variables of t

Abstract and concrete notation

Abstract notation:

- *a*, *b*, *c*, *d*, *e* for constants;
- *x*, *y*, *z*, *u*, *v*, *w* for variables;
- *f*, *g*, *h* for function symbols;
- *p*, *q* for relation symbols,

Example: *f*(*x*, *g*(*y*)).

Concrete notation:

Variable names start with upper-case letters.

Example: likes(john, Anybody).

Formulas

Atomic formulas, or atoms $p(t_1, \ldots, t_n)$.

$(A_1 \wedge \ldots \wedge A_n)$ and $(A_1 \vee \ldots \vee A_n)$

$(A \rightarrow B)$ and $(A \leftrightarrow B)$

$\neg A$

$\forall v\, A$ and $\exists v A$

Substitutions

Substitution θ *:* is any mapping from the set *V* of variables to the set of terms such that there is only a finite number of variables $v \in V$ with $\theta(v) \neq v$.

Domain $dom(\theta)$, range $ran(\theta)$ and variable range $vran(\theta)$:

$dom(\theta) = \{v \mid v \neq \theta(v)\}$,

$ran(\theta) = \{ t \mid \exists v \in dom(\theta)(\theta(v) = t)\}$,

$vran(\theta) = var(ran(\theta))$.

Notation: $\{ x_1 \mapsto t_1, \ldots , x_n \mapsto t_n \}$

empty substitution {}

Application of substitution

Application of a substitution θ to a term *t*:

- $x\theta = \theta(x)$
- $c\theta = c$
- $f(t_1, \ldots, t_n)\theta = f(t_1\theta, \ldots, t_n\theta)$

Herbrand interpretation

A Herbrand interpretation of a signature Σ is a set of ground atoms of this signature.

Truth in Herbrand Interpretations

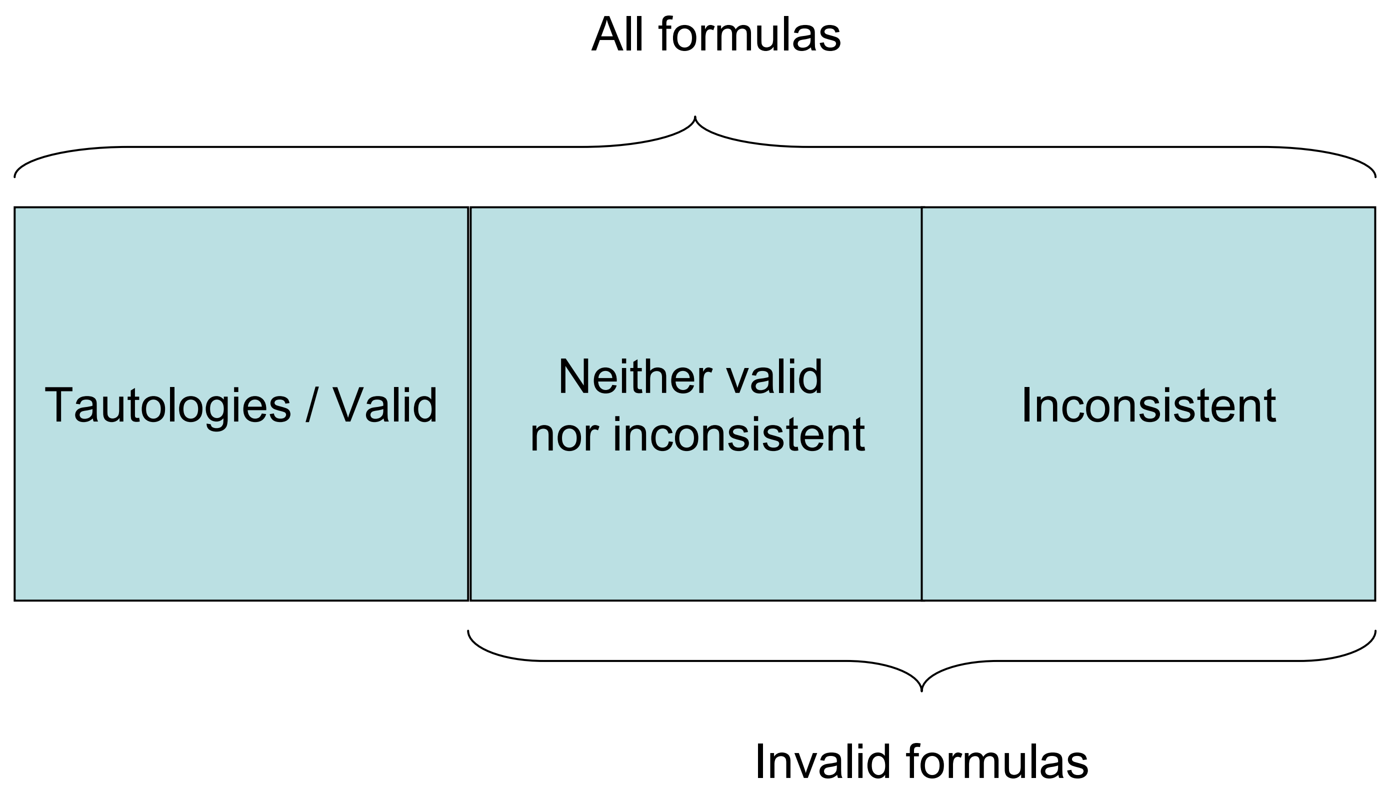
1. If A is atomic, then $I \models A$ if $A \in I$
2. $I \models B_1 \wedge \ldots \wedge B_n$ if $I \models B_i$ for all i
3. $I \models B_1 \vee \ldots \vee B_n$ if $I \models B_i$ for some i
4. $I \models B_1 \rightarrow B_2$ if either $I \models B_2$ or $I \not\models B_1$
5. $I \models \neg B$ if $I \not\models B$
6. $I \models \forall xB$ if $I \models B\{x \mapsto t\}$ for all ground terms t of the signature Σ
7. $I \models \exists xB$ if $I \models B\{x \mapsto t\}$ for some ground term t of the signature Σ

Tautology and Inconsistent

A formula F is a tautology (is valid), if $I \models F$ for every (Herbrand) interpretation I

A formula F is inconsistent, if $I \not\models F$ for every (Herbrand) interpretation I.

Types of Formulas



Logical Implication

A (set of) formula(s) F logically implies G (we write $F \models G$), iff every (Herbrand) interpretation I that fulfills F ($I \models F$) also fulfills G ($I \models G$).

$\wedge F \models G$ is true iff for every (Herbrand) interpretation I:
$I \models (\wedge F \rightarrow G)$

Literals

Literal is either an atom or the negation $\neg A$ of an atom A.

Positive literal: atom

Negative literal: negation of an atom

Complementary literals: A and $\neg A$

Notation: L

Clause

Clause: (or normal clause) formula $L_1 \wedge \ldots \wedge L_n \rightarrow A$,
where
$n \geq 0$, each L_i is a literal and A is an atom.

Notation: A :- $L_1 \wedge \ldots \wedge L_n$ or A :- $L_1, \ldots , L$

Head: the atom A.

Body: The conjunction $L_1 \wedge \ldots \wedge L_n$

Definite clause: all L_i are positive

Fact: clause with empty body

Syntactic Classification

Clause	Class
lives(Person, sweden) :- sells(Person, wine, Shop), not open(Shop,saturday)	normal
spy(Person) :- russian(Person)	definite
spy(bond)	fact

Goal

- Goal (also normal goal) is any conjunction of literals $L_1 \wedge \ldots \wedge L_n$
- Definite goal: all L_i are positive
- Empty goal □: when $n = 0$

Excercise:

- Syntax & Semantics for Monadic Fuzzy Logics

- Monadic: only unary predicates
- Fuzzy:
  - Truth values in [0,1]
  - operators for truth values
    - T-norm: $\top_{min}(a,b)=\min\{a,b\}$
    - T-conorm: $\bot_{min}(a,b)=\max\{a,b\}$
    - Also: $\top_{prod}(a,b)=a\cdot b$, $\bot_{sum}(a,b)=a+b-a\cdot b, \ldots$
    - Duality for $N(x)=1-x$