Stratified Programs
Observation:
Every normal program is consistent (has a model), but this is not necessarily true for \( \text{comp}(P) \).

Example:
program \( P \):

\[
\begin{align*}
    p & \leftarrow \neg q \\
    q & \leftarrow \neg r \\
    r & \leftarrow \neg p
\end{align*}
\]

\( I = \{ p, q, r \} \) is a model

\[
\begin{align*}
    p & \leftrightarrow \neg q \\
    q & \leftrightarrow \neg r \\
    r & \leftrightarrow \neg p
\end{align*}
\]

\( \text{comp}(P) \)

By transitivity:

\[
\begin{align*}
    p & \leftrightarrow \neg q \\
    q & \leftrightarrow \neg r \\
    r & \leftrightarrow \neg p
\end{align*}
\]

Thus there exists no model for \( \text{comp}(P) \)
Definition:

A *level mapping* of a normal program is a mapping from its set of predicate symbols to the non-negative integers. We refer to the value of a predicate symbol under this mapping as the *level* of that predicate symbol.
Level mapping:
  mapping ` from a set of relation symbols to N.
  l(r) is called the level of r.

**Theorem.** Let C be a finite non-recursive set of clauses. Then there exists a level mapping l such that for every clause c \( c \in C \),
  if q occurs in the body of c and c defines r,
  then \( l(r) > l(q) \).
Definition:
A normal program is hierarchical if it has a mapping such that in every program clause
$A \leftarrow L_1, \ldots, L_n$, the level of every predicate symbol occurring in the body is less than the level of $A$.

Observation:
not hierarchical:
relatedTo(x,y) $\leftarrow$ relatedTo(y,x)
Stratification

- **Definition:**
  A normal program is stratified if it has a level mapping such that in every clause $A \iff L_1, \ldots, L_n$,
  - the level of the predicate symbol of every positive literal is less or equal to the level of $A$ and
  - the level of each predicate symbol of every negative literal is less than the level of $A$. 
Example for Stratification

level 0
friend(a,b) ← 
friend(b,c) ← 

level 1
friend(x,z) ← friend(x,y), friend(y,z) 
enemy(x,y) ← friend(x,z), enemy(z,y), ~friend(x,y) 

level 2
loves(x,y) ← friend(x,y) 
loves(x,y) ← enemy(x,y) 

enemy(x,y) ← ~friend(x,y)
Counterexample

\[
\begin{align*}
\text{man}(x) &\leftarrow \text{person}(x), \neg \text{woman}(x) \\
\text{woman}(x) &\leftarrow \text{person}(x), \neg \text{man}(x). \\
\text{man}(x) &\leftarrow \text{person}(x), \neg \text{woman}(x). \\
\text{woman}(x) &\leftarrow \text{person}(x), \neg \text{man}(x). \\
\end{align*}
\]
**Corollary:**
Let $P$ be a stratified normal program. Then $\text{comp}(P)$ has a minimal normal Herbrand model. A normal Herbrand model assigns the equality relation to "=".
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Let $P$ be a stratified normal program. Then $\text{comp}(P)$ has a minimal normal Herbrand model. A normal Herbrand model assigns the equality relation to "$=\$".

Fixpoint for level 1

level 1

level 0
Corollary: Let \( P \) be a stratified normal program. then \( \text{comp}(P) \) has a minimal normal Herbrand model. A normal Herbrand model assigns the equality relation to "=".
Computational counterpart to models of stratified programmes:

Computing with finite failure
Definition
A normal program is locally stratified if each atom in $B_P$ can be assigned a countable ordinal level such that no atom positively depends on an atom of greater level negatively depends on an atom of equal or greater level.
Example for Local Stratification

even(s(X)) ← ¬even(X).
even(0).

Bₚ:
\{\text{even}(0)^0, \text{even}(s(0))^1, \text{even}(s(s(0)))^2,
\text{even}(\ldots)^3, \ldots\}
even(s(X)) ← ¬even(X).
even(0).
even(0) ← q(X).

\( B_p: \)
\( J= \{ q(0)^0, even(0)^1, even(s(s(0)))^3, \ldots \} \)
\( l= \{ even(0)^0, even(s(s(0)))^2, \ldots \} \)
Definition

Let P be a normal program and I a model. I is a perfect model for a given level of $B_P$, if
for every other model J,
if a positive literal p is the atom of least level in one model, but not in the other, then p is in J.

In other words, atoms of higher level are preferred for the perfect model.

Przymusinski: All locally stratified programs have a perfect model, which is independent of the ranking system chosen.