

## Relational Data Model

- Relational data model;
- Tuples and relations;
- Schemas and instances;
- Named vs. unnamed perspective;
- Relational algebra;

| Player | Birth Year |
|--------|------------|
| Andy   | 1980       |
| Wim    | 1975       |
| Liam   | 1985       |
| Mike   | 1988       |
| Bert   | 1971       |

- The **rows** of the table contain pairs occurring in the relation **player**.
- There are two **columns**, labeled respectively by “name” and “birth year”.
- The **values** in each column belong to different **domains** of possible values.

1. specifying the names of the columns (also called **fields** or **attributes**);
2. specifying a **domain** of possible values for each column;
3. enumerate all tuples in the relation.

(1)–(2) refer to the **schema** of this relation, while (3) to an **instance**.

- A set of **domains**  $D_1; D_2$  (sets of values);
- A set of corresponding **domain names**  $d_1, d_2, \dots$
- A set of **attributes**  $a_1, a_2, \dots$

**Tuple:** any finite sequence  $(v_1, \dots, v_n)$ .  
 $n$  is the **arity** of this tuple.

**Relation schema:**

$$r(a_1:d_1, \dots, a_n:d_n)$$

where  $n \geq 0$ ,  $r$  is a relation name,  $a_1, \dots, a_n$  are distinct attributes,  $d_1, \dots, d_n$  are domain names.

**Relation instance:**

finite set of tuples  $(v_1, \dots, v_n)$  of arity  $n$  such that  $v_i \in D_i$  for all  $i$ .

1. The attributes in each column must be unique.
2. A relation is a **set**. Therefore, when we represent a relation by a table, the order of rows in the table does not matter.

Let us add to this:

3. The order of attributes does not matter.



- A tuple is a set of pairs  $\{ (a_1, v_1), \dots, (a_n, v_n) \}$  denoted by  $\{ a_1=v_1, \dots, a_n=v_n \}$ ,
- Let  $d_1, \dots, d_n$  be domain names or  $D_1; \dots; D_n$  be the corresponding domains.
- The tuple **conforms** to a relation schema  $r(a_1:d_1, \dots, a_n:d_n)$  if  $v_i \in D_i$  for all  $i$ .

Note that in the relational data model tuples stored in a table are **structured**:

- all tuples conform to the same relation schema;
- the values in the same column belong to the same domain.

**Untyped perspective:** there is a single domain, so the second condition can be dropped.

- Consider the relation admire:

|         |                |
|---------|----------------|
| admirer | admired        |
| wim     | andy           |
| mike    | wim            |
| liam    | andy           |
| liam    | <b>arsenal</b> |

- Relational database schema: a collection of relation schemas with distinct relation names.
  
- Relational database instance conforming to a relational database schema  $S$ :
  - ◆ a mapping  $I$  from the relation names of  $S$  to relation instances such that
    - for every relation schema  $r(a_1:d_1, \dots, a_n:d_n)$  in  $S$  the relation instance  $I(r)$  conforms to this relation schema.

- **No attributes**
- a tuple is simply a **sequence**  $(v_1, \dots, v_n)$  of values.
- The components of tuples can therefore be identified by their **position** in the tuple.

- Introduce a collection of attributes  $\#1, \#2, \dots,$
- identify tuple  $(v_1, \dots, v_n)$  with the tuple  $\{ \#1 = v_1, \dots, \#n = v_n \}$ .
- Likewise, identify relation schema  $r(d_1, \dots, d_n)$  with  $r(\#1:d_1, \dots, \#n:d_n)$ .

1. Can **define** new relations from existing ones;
2. Uses a collection of **operations** on relations to do so.

$$\begin{aligned} & \{ (v_{11}, \dots, v_{1n}), \\ & \dots \\ & (v_{k1}, \dots, v_{kn}) \} \end{aligned}$$



$$R_1 \hat{\cup} R_2 = \{(c_1, \dots, c_k) \mid (c_1, \dots, c_k) \in R_1 \text{ or } (c_1, \dots, c_k) \in R_2\}$$

$$R_1 - R_2 =$$

$$\{(c_1, \dots, c_k) \mid (c_1, \dots, c_k) \in R_1 \text{ and } (c_1, \dots, c_k) \notin R_2\}$$

$R_1 \times R_2 =$

$\{(c_1, \dots, c_k, d_1, \dots, d_m) \mid$

$(c_1, \dots, c_k) \in R_1 \text{ and}$

$(d_1, \dots, d_m) \in R_2 \}.$

Let now  $R$  be a relation of arity  $k$  and  $i_1, \dots, i_m$  be numbers in  $\{1, \dots, k\}$ .

$$\pi_{i_1, \dots, i_m}(R) = \{(c_{i_1}, \dots, c_{i_m}) \mid (c_1, \dots, c_k) \in R\}.$$

We say that  $\pi_{i_1, \dots, i_m}(R)$  is obtained from  $R$  by **projection** (on arguments  $i_1, \dots, i_m$ ).

Assume **formulas on domains** with “variables” #1, #2, ....

For example, #1 = #2.

$$\hat{I}(R) = \{(c_1, \dots, c_k) \mid \\ (c_1, \dots, c_k) \in R \text{ and} \\ F \text{ holds on } (c_1, \dots, c_k)\}.$$

- Relational algebra, named perspective
- SQL
- Integrity constraints
- (Aggregates and grouping)

$$\begin{aligned} & \{ \{ a_1 = v_{11}, \dots, a_n = v_{1n} \}, \\ & \quad \dots \quad \dots \quad \dots \\ & \{ a_1 = v_{k1}, \dots, a_n = v_{kn} \} \} \end{aligned}$$

Let  $R_1, R_2$  be relations with the same attributes.

$$R_1 \hat{\cup} R_2 = \{ t \mid t \in R_1 \text{ or } t \in R_2 \}$$



$R_1$

| A | B |
|---|---|
| - | 1 |
| - | 2 |
| † | 1 |

$R_2$

| A | B |
|---|---|
| - | 2 |
| † | 3 |

$R_1 \hat{=} R_2$

| A | B |
|---|---|
| - | 1 |
| - | 2 |
| † | 1 |
| † | 3 |

Let  $R$  be a relation whose set of attributes is  $a_1, \dots, a_n, c_1, \dots, c_m$

Let  $b_1, \dots, b_n$  be distinct attributes such that

$$\{b_1, \dots, b_n\} \sim \{c_1, \dots, c_m\} = >$$

Then

$$\rho_{d_1 \rightarrow e_1, \dots, d_n \rightarrow e_n} (R) = \\ \{ \{b_1 = v_1, \dots, b_n = v_n, c_1 = w_1, \dots, c_m = w_m\} \mid \\ \{a_1 = v_1, \dots, a_n = v_n, c_1 = w_1, \dots, c_m = w_m\} \in R \}$$

SQL is based on set and relational operations with certain modifications and enhancements

A typical SQL query has the form:

**select**  $a_1, \dots, a_n$   
**from**  $R_1, \dots, R_m$   
**where**  $P$

This query is equivalent to relational algebra expression:

$$\sigma_{d_1, \dots, d_q} (\rho_s (R_1 \bowtie \dots \bowtie R_m))$$

The result of an SQL query is a relation.

Exceptions?

- Domain constraints.
- Key constraints.
- Foreign key constraints.
  
- More general, defined constraints.
  
- How to translate them?

Allow one to define:

- Relation and database schemas;
- Relations through our relations;
- Integrity constraints;
- Updates.