# Models in First Order Logics

People and Knowledge Networks

First-order logic. Syntax and semantics.

Herbrand interpretations;

Clauses and goals;

Datalog.

# First-order signature

First-order signature « consists of

- *con* — the set of constants of « ;
- *fun* — the set of function symbols of « ;
- *rel* — the set of relation symbols of « .

# Terms

Term of « with variables in *X*:

1. Constant *c* 5 *con*;
2. Variable *v* 5 *X*;
3. If *f* 5 *fun* is a function symbol of arity *n* and $t_1$, … , $t_n$ are terms, then $f(t_1, \ldots, t_n)$ is a term.

A term is ground if it has no variables

*var*(*t*) — the set of variables of *t*

# *Abstract and concrete notation*

Abstract notation:

- *a, b, c, d, e* for constants;
- *x, y, z, u, v, w* for variables;
- *f, g, h* for function symbols;
- *p, q* for relation symbols,

Example: *f*(*x*, *g*(*y*)).

Concrete notation:

Variable names start with upper-case letters.

Example: likes(john, Anybody).

# *Formulas*

Atomic formulas, or atoms $p(t_1, \ldots, t_n)$.

($A_1$ – … – $A_n$) and ($A_1$ Æ … Æ $A_n$)

(A $ B) and (A ′ B)

= A

;v A and <vA

Substitution $\sigma$ : is any mapping from the set $V$ of variables to the set of terms such that there is only a finite number of variables $v \in V$ with $\sigma(v) \neq v$.

Domain $dom(\sigma)$, range $ran(\sigma)$ and variable range $vran(\sigma)$:

$$dom(\sigma) = \{v \mid v \neq \sigma(v)\},$$

$$ran(\sigma) = \{ t \mid \exists v \in dom(\sigma)(\sigma(v) = t)\},$$

$$vran(\sigma) = var(ran(\sigma)).$$

Notation: $\{ x_1 \mapsto t_1, \ldots , x_n \mapsto t_n \}$

empty substitution {}

# *Application of substitution*

Application of a substitution „ to a term *t*:

- $x$„ = „($x$)
- $c$„ = $c$
- $f(t_1, \dots, t_n)$„ = $f(t_1$„, … , $t_n$„)

# *Herbrand interpretation*

A Herbrand interpretation of a signature « is a set of ground atoms of this signature.

# *Truth in Herbrand Interpretations*

1. If A is atomic, then I ¶ A if A 5 I

2. I ¶ $B_1$ – … – $B_n$ if I ¶ $B_i$ for all i

3. I ¶ $B_1$ Æ … Æ $B_n$ if I ¶ $B_i$ for some i

4. I ¶ $B_1$ $ $B_2$ if either I ¶ $B_2$ or I ¶̸ $B_1$

5. I ¶ = B if I ¶̸ B

6. I ¶ ; xB if I ¶ B{x ↦ t} for all ground terms t of the signature «

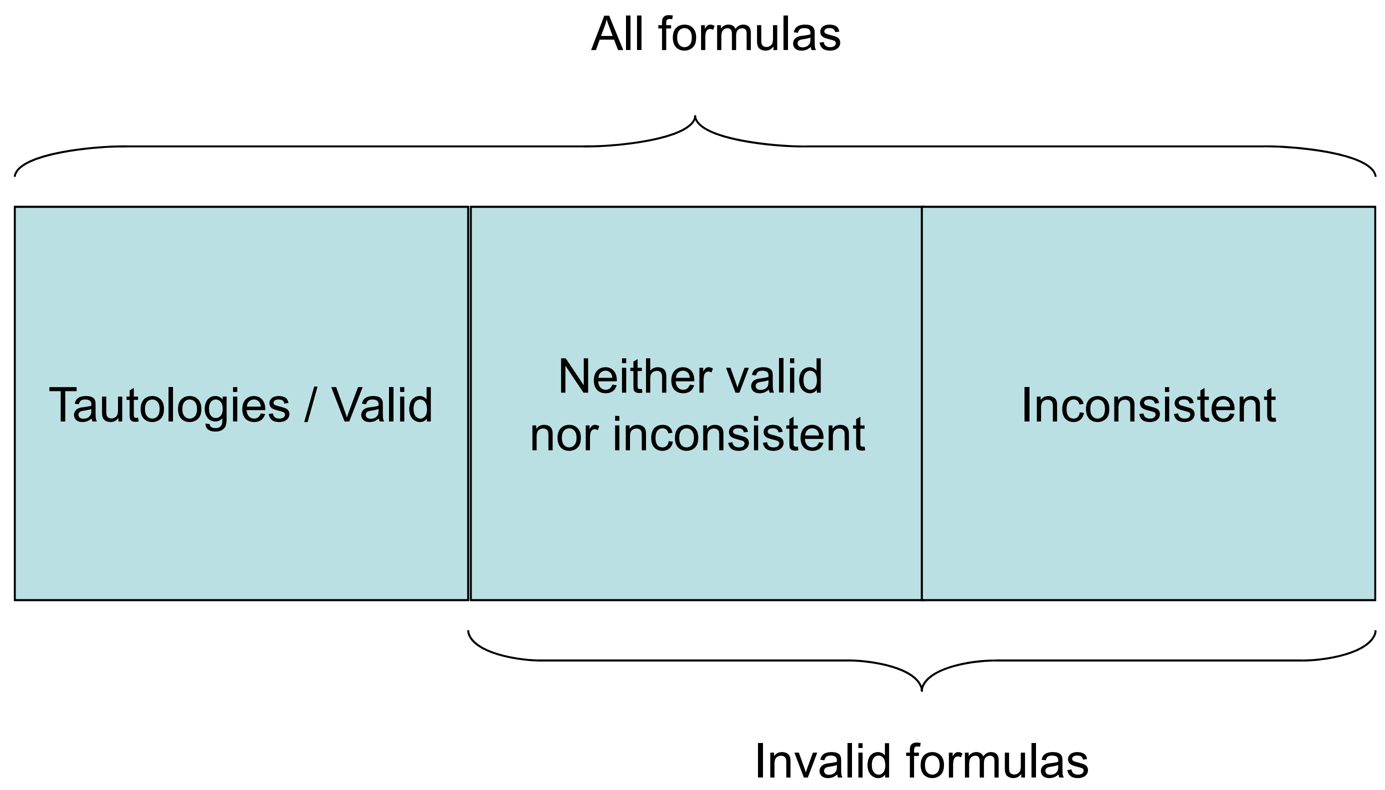
7. I ¶ <xB if I ¶ B{x ↦ t} for some ground term t of the signature «

# Tautology and Inconsistent

A formula F is a tautology (is valid), if I ¶ F for every (Herbrand) interpretation I

A formula F is inconsistent, if I /¶ F for every (Herbrand) interpretation I.

# Types of Formulas



# Logical Implication

A (set of) formula(s) F logically implies G (we write F ¶ G), iff every (Herbrand) interpretation I that fulfills F (I ¶ F) also fulfills G (I ¶ G).

–F ¶ G is true iff for every (Herbrand) interpretation I:
I ¶ (–F $ G)

# *Literals*

Literal is either an atom or the negation $=A$ of an atom $A$.

Positive literal: atom

Negative literal: negation of an atom

Complementary literals: $A$ and $=A$

Notation: L

Clause: (or normal clause) formula $L_1$ – … – $L_n$ $ A,
where
n ¿ 0, each $L_i$ is a literal and A is an atom.

Notation: A :- $L_1$ – … – $L_n$ or A :- $L_1$, … , L

Head: the atom A.

Body: The conjunction $L_1$ – … – $L_n$

Definite clause: all $L_i$ are positive

Fact: clause with empty body

# *Syntactic Classification*

| Clause | Class |
|---|---|
| lives(Person, sweden) :- sells(Person, wine, Shop), not open(Shop,saturday) | normal |
| spy(Person) :- russian(Person) | definite |
| spy(bond) | fact |

# Goal

- Goal (also normal goal) is any conjunction of literals $L_1 - \ldots - L_n$
- Definite goal: all $L_i$ are positive
- Empty goal □: when $n$ = 0

Excercise:

- Syntax & Semantics for Monadic Fuzzy Logics

- Monadic: only unary predicates
- Fuzzy:
  - Truth values in [0,1]
  - operators for truth values
    - T-norm: $\mathrm{A}_{min}(a,b)=\min\{a,b\}$
    - T-conorm: $\mathrm{B}_{min}(a,b)=\max\{a,b\}$
    - Also: $\mathrm{A}_{prod}(a,b)=a ∫ b$, $\mathrm{B}_{sum}(a,b)=a+b-a ∫ b$,…
    - Duality for $N(x)=1-x$