Stable Models

See Bry et al 2007

p :- not p.

Justification postulate

- Requests dependable justifications for derived truths.

⇒ Some programs do not have a model (cf above)

Consistency postulate

- Every syntactically correct set of normal clauses is consistent and must therefore have a model.

⇒ {p} is a model for the program above

Controversy

p :- not q.
q :- not p

Justification postulate

- Requests dependable justifications for derived truths.

⇒ Both {p} and {q} are reasonable models

Consistency postulate

- Every syntactically correct set of normal clauses is consistent and must therefore have a model.

Definition: ***Gelfond-Lifschitz transformation***

Let S be a (possibly infinite) set of ground normal clauses, i.e. of formulas of the form

$$A \text{ :- } L_1, \ldots., L_n$$

where $n \geq 0$ and A is a ground atom and the L_i are ground literals. Let $B \subseteq B_P$.

The Gelfond-Lifschitz transform $GL_B(S)$ of S with respect to B is obtained from S as follows:

1. remove each clause whose antecedent contains a literal $\neg A$ with $A \in B$.
2. remove from the antecedents of the remaining clauses all negative literals.

Example Gelfond-Lifschitz transformation

Program:

brother(X,Y) :- brother(X,Z),brother(Z,Y), not =(X,Y).

brother(chico,harpo).

brother(harpo,chico).

****

Grounded Program:

brother(chico,chico) :- brother(chico,harpo),brother(harpo,chico), not =(chico,chico)

brother(chico,harpo) :- brother(chico,chico),brother(chico,harpo), not =(chico,harpo)

…[5 more]…

brother(harpo,harpo) :- brother(harpo,chico), brother(chico,harpo), not =(harpo,harpo)

brother(chico,harpo).

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Example Gelfond-Lifschitz transformation

S={

brother(chico,chico) :- brother(chico,harpo),brother(harpo,chico), not =(chico,chico)

brother(chico,harpo) :- brother(chico,chico),brother(chico,harpo), not =(chico,harpo)

…[5 more]…

brother(harpo,harpo) :- brother(harpo,chico), brother(chico,harpo), not =(harpo,harpo)

brother(chico,harpo).

brother(harpo,chico).

}

Ex 1: B={brother(chico,harpo), brother(harpo,chico), =(chico,chico), =(harpo,harpo)}

GL_B(S)={

~~**brother(chico,chico) :- brother(chico,harpo),brother(harpo,chico), not =(chico,chico)**~~

brother(chico,harpo) :- brother(chico,chico),brother(chico,harpo), not =(chico,harpo)

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…[5 more]…

~~**brother(harpo,harpo) :- brother(harpo,chico), brother(chico,harpo), not =(harpo,harpo)**~~

brother(chico,harpo), brother(harpo,chico). }

Stable model semantics

Definition (stable model):

Let S be a (possibly infinite) set of ground normal clauses. An Herbrand interpretation B is a stable model of S, **iff** it is the unique minimal Herbrand model of $GL_B(S)$.

Note:

A stable model of a set S of normal clauses is a stable model of the (possibly infinite) set of ground instances of S.

Lemma: Let S be a set of ground normal clauses and B an Herbrand interpretation. $B \models S$ iff $B \models GL_B(S)$

Stable model semantics

Definition (stable model):

Let S be a (possibly infinite) set of ground normal clauses. An Herbrand interpretation B is a stable model of S, **iff** it is the unique minimal Herbrand model of $GL_B(S)$.

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A stable model of a set S of normal clauses is a stable model of the (possibly infinite) set of ground instances of S.

Lemma: Let S be a set of normal clauses. Each stable model of S is a minimal Herbrand model of S.

Examples

$S_1 = \{$

(p :- not p),

(p :- true)}

Has the stable model {p}.

$GL_{\{p\}}(S) = \{(p\text{:-}true)\}$, which has the unique minimal model {p}

It has no other model.

Examples

S_2 = {

(p :- not p)}

has no stable model.

It has the model {p}, but $GL_{\{p\}}$ (S) = {}, which has the unique minimal model {}

It has the model {}, but $GL_{\{p\}}$ (S) = { (p:-true)}, which has the unique minimal model {p}

S_3 = {

(q :- r, not p),

(r :- s, not t),

(s :- true)}

Has the following models:

- {s,r,q}, {s,t,q}, {s,t,p},…

But after applying $GL_B(S)$ p and t cannot be part of the unique minimal model and {s,r,q} must be!

Therefore it has the single stable model {s,r,q}

Examples

S_4 = {

(q :- not p),
(p :- not q)}

Has the following models:

- {q}, {p}

Both are stable models!

Cautious vs brave (skeptical vs credolous)

Logical consequence in stable model semantics

- Cautious (skeptical) entailment:
  - P |= F, iff F is true in all stable models of P
- Brave (credulous) entailment:
  - P |= F, iff F is true in some stable model of P
- Main interest typically:
  - The different models with their different properties

Observations on stable models

- Stable model semantics coincides with the intuitive understanding based on the „justification postulate“.

- Unintuitive minimal models of the examples turn out not to be stable and the stability criterion retains only those minimal modes that are intuitive.

- A set may have several stable models or exactly one or none

- Each stratifiable set has exactly one stable model.

Example well-founded model

S_1 = {

(p :- not p),

(p :- true)}

Has the well-founded model ({p},{})

Example well-founded model

S_2 = {

(p :- not p)}

has the well founded model ({},{})

Examples

S_3 = {

(q :- r, not p),

(r :- s, not t),

(s :- true)}

Has the well-founded model ({s,r,q},{t,p})

Examples

S_4 = {

(q :- not p),

(p :- not q)}

Has the well-founded model ({},{})

Comparison

Stable model semantics

- Justification postulate

Well-founded semantics

- Always one model / consistency postulate

If a rule set is stratifiable, then it has a unique minimal model, which is a stable model and at the same time a total well-founded model

If a rule set S has a total well-founded model, then this model is also the single stable model of S and vice versa.

If a rule set S has a partial well-founded model I that is not total, then S has either no stable model or more than one. In this case a ground atom is true (or false, respectively) in all stable models of S if and only if it is true in I (or false, respectively).

Comparison

Stable model semantics

- Justification postulate

Well-founded semantics

- Always one model / consistency postulate

Well-founded semantics convey the „agreement“ of stable models.

Well-founded semantics cannot distinguish between several justifiable models (S_4) and no justifiable model (S_2)

Side remark

p :- odd(X), not odd(X).

odd(s(X)) :- not odd(X).

Well founded model is:

I_{2n} = ({odd(s(0)), odd(s(s(s(0)))),….,odd(s^{2n-1}(0)},
{odd(0),odd(s(s(0))),… ,odd(s^{2n-2}(0))})

Fixpoint: $I_{\omega+1} = I_\omega \cup (\{\},\{p\})$

d.h. ¬p

WFS is undecidable, NAF is semi-decidable