1 Relational vs. Deductive Data Model (3 points)

Exemplify that non-recursive Datalog has the same expressive power as relational algebra. Give for the relational operations (cf. Slides No. 2) the corresponding rules in Datalog (or Prolog).

2 Substitution (4 points)

Specify a substitution $\sigma$ such that the terms on the left side become equal to the terms on the right side. Give the reasons if there is no substitution.

1. $g(a, X, Y, b)$ and $g(a, a, h(Y), b)$
2. $g(a, X, h(Y), b)$ and $g(a, h(Z), h(Z), b)$
3. $g(a, Y, f(Z, b), c)$ and $g(a, h(c), f(a, X), c)$
4. $g(a, h(X), f(X, b), c)$ and $g(a, h(h(Z)), f(d, b), c)$
5. $f(X, X)$ and $f(h(Y), h(a))$
6. $h(g(X, f(a, Y), c, h(Z)))$ and $h(g(b, f(a, h(Y)), c, h(Y)))$

3 Interpretation (3 points)

Assume the following interpretation $I$:

- The domain is $\mathbb{N}_0$
- $a$ is a constant assigned to 0, and the constant $b$ is assigned to 1
- The predicate $g(X, Y)$ is assigned to the relation $\{(X, Y) : X > Y\}$
- $s$ is the successor function $x \rightarrow x + 1$

Determine whether the following formulas are true or false:

1. $\forall X \exists Y g(X, Y)$
2. $\forall X g(s(X), X)$

3. $\forall X \exists Y \exists Z (g(X, s(Y)) \land g(s(Z), X))$

4. $\forall X \forall Y \exists Z (g(X, s(Y)) \rightarrow g(Z, s(Y)))$