Semantic Web

2. Description logics (2/2)

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Recap

- Ontologies: definition and applications
- Structure of an ontology
- The description logic $\mathcal{ALC}$:
  - Syntax: signature, concepts, relations, axioms
  - Semantic: interpretations, models
  - Inference: entailment
  - $\mathcal{ALC}$ and first-order logics

Recap

- Ontologies: definition and applications
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  - Syntax: signature, concepts, relations, axioms
  - Semantic: interpretations, models
  - Inference: entailment
  - $\mathcal{ALC}$ and first-order logics

Home assignment:
Consider $\mathcal{K}_2 = (\mathcal{T}_2, \mathcal{A}_2)$ given via

$$\begin{aligned} \mathcal{T}_2 = \{ \quad & B \sqsubseteq D \quad \} \\ \mathcal{A}_2 = \{ \quad & d : B, (c, d) : S \quad \} \end{aligned}$$

Is the entailment $\mathcal{K}_2 \models c : \forall S.D$ valid? (Home assignment)

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- Today we have a look at a very simple proof procedure for deciding consistency: the *Tableau Algorithm*

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- Formal syntax and semantics provide the basis for understanding description logics
- Implementing the semantics of e. g. $\mathcal{ALC}$ is intractable for obtaining a proof procedure
  - Generate all interpretations
  - Check whether an interpretation satisfies a potential conclusions
- Today we have a look at a very simple proof procedure for deciding consistency: the *Tableau Algorithm*
- We also take another look at ontology languages in general and applications

Outline

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Inference tasks 1/2

In description logics one usually distinguishes the following *inference tasks*:

- Subsumption problem: Given concepts C_1, C_2 does $\mathcal{K}$ entail $C_1 \sqsubseteq C_2$?
- Instance checking problem: Given concept C and individual t does $\mathcal{K}$ entail $t : C$?
- Relation checking problem: Given relation R and individuals t, t' does $\mathcal{K}$ entail $(t, t') : R$?
- Consistency problem: Is there $\mathcal{I}$ with $\mathcal{I} \models \mathcal{K}$?

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Inference tasks 2/2

The subsumption and the consistency problem are closely related:

- C_2 subsumes C_1 if C_1 and $\neg C_2$ are inconsistent:

$$\mathcal{K} \models C_1 \sqsubseteq C_2 \quad \text{if and only if} \quad \neg\exists\mathcal{I} : \mathcal{I} \models \mathcal{K} \cup \{t : (C_1 \sqcap \neg C_2)\}$$

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$\rightarrow$ it suffices to investigate algorithms for checking consistency.

The Tableau Algorithm

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- If this is successful, $\mathcal{K}$ is consistent
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- $\mathcal{S}$ is initialized with the singleton $\mathcal{S} = \{\mathcal{A}\}$
- Apply different rules on the elements of $\mathcal{S}$ depending on the axioms in $\mathcal{T}$
- If no more rules are applicable, $\mathcal{K}$ is consistent if there is an ABox $\mathcal{A}'$ in $\mathcal{S}$ that is consistent (contains no axioms $t : C$, $t : \neg C$)

The Tableau Algorithm - Negation Normal Form

In order to apply the tableau algorithm we have to assume that $\mathcal{K}$ is in *negation normal form*:

- $\neg(C_1 \sqcup C_2) \quad \longrightarrow \quad \neg C_1 \sqcap \neg C_2$
- $\neg(C_1 \sqcap C_2) \quad \longrightarrow \quad \neg C_1 \sqcup \neg C_2$
- $\neg\exists R.C \quad \longrightarrow \quad \forall R.(\neg C)$
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From now on, we assume that every concept appearing in an axiom in $\mathcal{K}$ is in negation normal form. For example

$$\neg(C_1 \sqcap C_2) \sqsubseteq \neg\exists R.\neg(C_3 \sqcup C_4) \quad \longrightarrow \quad \neg C_1 \sqcup \neg C_2 \sqsubseteq \forall R.(C_3 \sqcup C_4)$$

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- $\sqcap$-rule: if $t : C_1 \sqcap C_2 \in \mathcal{A}'$ and $\{t : C_1, t : C_2\} \not\subseteq \mathcal{A}'$ then remove $\mathcal{A}'$ from $\mathcal{S}$ and add $\mathcal{A}' \cup \{t : C_1, t : C_2\}$ to $\mathcal{S}$.

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- $\sqcup$-rule: if $t : C_1 \sqcup C_2 \in \mathcal{A}'$ and $\{t : C_1, t : C_2\} \cap \mathcal{A}' = \emptyset$ then remove $\mathcal{A}'$ from $\mathcal{S}$ and add both $\mathcal{A}' \cup \{t : C_1\}$ and $\mathcal{A}' \cup \{t : C_2\}$ to $\mathcal{S}$.

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- $\exists$-rule: if $t : \exists R.C \in \mathcal{A}'$ and there is no t' with $\{(t, t') : R, t' : C\} \subseteq \mathcal{A}'$ then remove $\mathcal{A}'$ from $\mathcal{S}$, *create* a new individual t'', and add $\mathcal{A}' \cup \{(t, t'') : R, t'' : C\}$ to $\mathcal{S}$.

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- $\forall$-rule: if $\{t : \forall R.C, (t, t') : R\} \subseteq \mathcal{A}'$ and $\{t' : C\} \notin \mathcal{A}'$ then remove $\mathcal{A}'$ from $\mathcal{S}$ and add $\mathcal{A}' \cup \{t' : C\}$ to $\mathcal{S}$.

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- $\sqsubseteq$-rule: if $C_1 \sqsubseteq C_2 \in \mathcal{T}$ and $t : (\neg C_1 \sqcup C_2) \notin \mathcal{A}'$ for t appearing in $\mathcal{A}'$ then remove $\mathcal{A}'$ from $\mathcal{S}$ and add $\mathcal{A}' \cup \{t : (\neg C_1 \sqcup C_2)\}$ to $\mathcal{S}$.

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The Tableau Algorithm - Approach

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ and $\mathcal{A}$ be consistent.

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The tableau algorithm:

1. Set $\mathcal{S} = \{\mathcal{A}\}$
2. Is some rule applicable?
   - yes: goto 3
   - no: $\mathcal{K}$ is consistent; exit
3. Apply the rule to $\mathcal{S}$
4. Remove all $\mathcal{A}' \in \mathcal{S}$ with $t : C, t : \neg C \in \mathcal{A}'$ (for some t, C)
5. $\mathcal{S} = \emptyset$?
   - yes: $\mathcal{K}$ is inconsistent; exit
   - no: goto 2

The Tableau Algorithm - Example

Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be given via

$$\mathcal{T} = \{ \quad A \sqsubseteq \exists R.C \quad \}$$
$$\mathcal{A} = \{ \quad a : A, b : D, (a, b) : R \quad \}$$

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Let $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ be given via

$$\begin{aligned} \mathcal{T} &= \{ \quad A \sqsubseteq \exists R.C \quad \} \\ \mathcal{A} &= \{ \quad a : A, b : D, (a, b) : R \quad \} \end{aligned}$$

Observe:

- $\mathcal{K}$ is in negation normal form
- $\mathcal{A}$ is consistent

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Observe:

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Initialize $\mathcal{S} = \{\{a : A, b : D, (a, b) : R\}\}$.

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The Tableau Algorithm - Example cont'd

$$\mathcal{T} = \{ \quad A \sqsubseteq \exists R.C \quad \}$$

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- All ABoxes in $\mathcal{S}$ are consistent

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$\rightarrow$ The knowledge base $\mathcal{K}$ is consistent.

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What about termination?

The Tableau Algorithm - Blocking Rules

What happens when applying the tableau algorithm to

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To ensure termination we introduce the notion of *block*. If

- t is an individual created by application of a rule and
- there is an individual t' with
  1. $\{C \mid t : C \in \mathcal{A}\} \subseteq \{C \mid t' : C \in \mathcal{A}\}$ and
  2. t' has been created before t

then t is blocked (by t').

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The Tableau Algorithm - Blocking Rules cont'd

- $\sqcap$-rule: if $t : C_1 \sqcap C_2 \in \mathcal{A}'$, t **is not blocked**, and $\{t : C_1, t : C_2\} \not\subseteq \mathcal{A}'$ then remove $\mathcal{A}'$ from $\mathcal{S}$ and add $\mathcal{A}' \cup \{t : C_1, t : C_2\}$ to $\mathcal{S}$.
- $\sqcup$-rule: if $t : C_1 \sqcup C_2 \in \mathcal{A}'$, t **is not blocked**, and $\{t : C_1, t : C_2\} \cap \mathcal{A}' = \emptyset$ then remove $\mathcal{A}'$ from $\mathcal{S}$ and add both $\mathcal{A}' \cup \{t : C_1\}$ and $\mathcal{A}' \cup \{t : C_2\}$ to $\mathcal{S}$.
- $\exists$-rule: if $t : \exists R.C \in \mathcal{A}'$, t **is not blocked**, and there is no t' with $\{(t, t') : R, t' : C\} \subseteq \mathcal{A}'$ then remove $\mathcal{A}'$ from $\mathcal{S}$, *create* a new individual t'', and add $\mathcal{A}' \cup \{(t, t'') : R, t'' : C\}$ to $\mathcal{S}$.
- $\forall$-rule: if $\{t : \forall R.C, (t, t') : R\} \subseteq \mathcal{A}'$, t **is not blocked**, and $\{t' : C\} \notin \mathcal{A}'$ then remove $\mathcal{A}'$ from $\mathcal{S}$ and add $\mathcal{A}' \cup \{t' : C\}$ to $\mathcal{S}$.
- $\sqsubseteq$-rule: if $C_1 \sqsubseteq C_2 \in \mathcal{T}$, t **is not blocked**, and $t : (\neg C_1 \sqcup C_2) \notin \mathcal{A}'$ for t appearing in $\mathcal{A}'$ then remove $\mathcal{A}'$ from $\mathcal{S}$ and add $\mathcal{A}' \cup \{t : (\neg C_1 \sqcup C_2)\}$ to $\mathcal{S}$.

The Tableau Algorithm - Blocking Rules cont'd

What happens now to

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- *The tableau algorithm with blocking always terminates on* $\mathcal{ALC}$.

The Tableau Algorithm - Blocking Rules cont'd

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- *The tableau algorithm with blocking always terminates on $\mathcal{ALC}$.*
- *The tableau algorithm with blocking runs in EXPSPACE (worst case).*

Outline

Ontology languages revisited

- $\mathcal{ALC}$ is just one example of a description logic

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- Over the years a lot of different description logics have been proposed that differ in
  - complexity
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- The search for the *right* description logic is ongoing
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  - $\mathcal{H}$: role hierarchies: $R \circ R' \subseteq R''$ $(\mathit{hasParent} \circ \mathit{hasParent} \equiv \mathit{hasGrandparent})$
  - ...

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- Some description languages with further restrictions:
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  - $\mathcal{EL}++$: $\mathcal{EL}$ with nominals and some additional role operators

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Outline

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- In contrast to many other KR languages (propositional logic, default logic, ...) description logics have been developed out of the need to apply them

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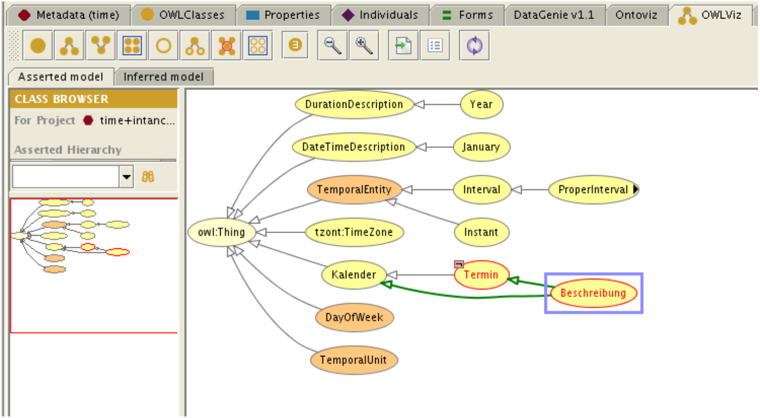
- In contrast to many other KR languages (propositional logic, default logic, ...) description logics have been developed out of the need to apply them
- Therefore, a lot of tools are around that enable ontology engineering using description logics

Tools: Protégé



- Free open-source ontology editor
- Webpage: `http://protege.stanford.edu`

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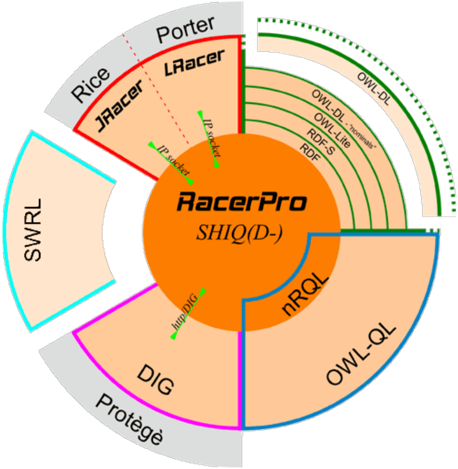
Tools: RacerPro



- Commercial description logic reasoner
- Webpage: `http://www.franz.com/agraph/racer`

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OWL API

- Official open source JAVA interfaces for programming DL applications
- Webpage: `http://owlapi.sourceforge.net`

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Outline

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Summary

- The tableau algorithm for $\mathcal{ALC}$:
  - checks consistency of a knowledge base
  - sound and complete
  - terminates always when using blocks
- Ontology languages revisited
  - Nomenclature of description logics
  - expressivity vs. complexity
- Tools for working with description logics

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Pointers to further reading

- The Description Logic Complexity Navigator
  http://www.cs.man.ac.uk/~ezolin/dl/
- Franz Baader. Tableau Algorithms for Description Logics.
  http://lat.inf.tu-dresden.de/~baader/Talks/Tableaux2000.pdf
- Franz Baader, Ulrike Sattler. An Overview of Tableau Algorithms for Description Logics. http://www.eecs.yorku.ca/course_archive/2010-11/F/6390A/DLmaterial/BaaderSattler-tableauxForDL.pdf

Exercises

- Apply the tableau algorithm to check whether the knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ is consistent:

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- Apply the tableau algorithm with blocking to check whether the knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ is consistent:

$$\mathcal{T} = \{ \quad A \sqsubseteq \exists R.B, \quad B \sqsubseteq A \sqcap \forall S.C \quad \}$$
$$\mathcal{A} = \{ \quad a : A \quad \}$$

(Home assignment)