Advanced Data Modeling

Summer Semester 2008
- Exercises III -

To be handed in before ***2008-05-17, 23:59*** *via e-mail to*
sschenk@uni-koblenz.de, subject line: [ADM] ...

1) The Fibonacci Numbers are computed as follows:
The 0^{th} Fibonacci Number is 0. The 1^{st} Fibonacci Number is 1. The n^{th} Fibonacci Number is the sum of the $n\text{-}1^{st}$ and the $n\text{-}2^{nd}$ Fibonacci Number for n>1.
Write a logic program with a predicate fib/2, which computes Fibonacci Numbers, such that the goal fib(n, X) binds X to the n^{th} Fibonacci Number. Model the Natural Numbers using only the constant 0 and the successor function f. Do not use any builtin arithmetic functions, but model the necessary operations yourself, for example using a predicate sum(X,Y,Z) for X+Y=Z.

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fib(0,0).
fib(f(0),f(0)).
fib(f(f(X)),Y) :- sum(V,W,Y), fib(X,V), fib(f(X),W).
sum(X,0,X).
sum(X,f(Y),Z) :- sum(f(X),Y,Z).
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2) The factorial of a Natural Number n is $\prod_{i=1}^{n} i$. Write a program computing the factorial of a natural number without using any builtin arithmetic functions. You may reuse predicates from 1).

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fact(f(0), f(0)).
fact(f(X),Y) :- prod(f(X),Z,Y), fact(X,Z).
prod(X,0,0).
prod(X,f(0),X).
prod(X,f(Y),Z) :- sum(X, W, Z), prod(X,Y,W).
```

1. List the Herbrand universe of your program.

   {0, f(0), f(f(0)), ...}

2. List the Herbrand base of your program.

   {fact(0,0), fact(0,f(0)), fact(0,f(f(0))), ...,
   fact(f(0),0), fact(f(0),f(0)), fact(f(0),f(f(0))), ...,
   prod(0,0,0), prod(0,f(0),0), prod(0,f(f(0)),0), ...,
   prod(f(0),0,0), prod(f(0),f(0),0), prod(f(0),f(f(0)),0), ...,
   sum(0,0,0), sum(0,f(0),0), sum(0,f(f(0)),0), ...,
   sum(f(0),0,0), sum(f(0),f(0),0), sum(f(0),f(f(0)),0), ...}

3. Does your program have a smallest Herbrand model?

   Yes, it is a definite program and hence has a smallest herbrand model.

4. If yes, describe the smallest Herbrand model of your program.

   It contains exactly the pairs of all natural numbers and their factorials, of all pairs of natural numbers and their sums and products.

3) What is the difference between an interpretation in general and an Herbrand interpretation?

   An Herbrand interpretation maps every constant to itself, while in general an interpretation maps every constant to some, possibly real-world, object.

4) What is the difference between an interpretation and a model?

   An interpretation assigns a truth value to every element of the Herbrand base. A model is an interpretation, in which every clause of the program is true.

5) Prove that every definite program has a minimal Herbrand model.

   Forward chaining from the facts of the program leads exactly to the minimal Herbrand model. As the program is definite, we can never come up with a contradiction.

6) Prove that every normal program has a minimal Herbrand model.

   This is not the case. As a counter example, take program 1. from 7). This program has no (two-valued) minimal model.

7) Which of the following programs are factual (definite, normal)? Which ones have minimal Herbrand models?

   1. p(a).
      m(X) :- p(X), not w(X)
      w(X) :- p(X), not m(X)

      Normal,
      no minimal model

   2. p(a).
      w(a).
      m(X) :- p(X), not w(X)
      w(X) :- p(X), not m(X)

      Normal,
      minimal model: {p(a), w(a)}

   3. p(a).
      w(a).
      m(b).
      m(b) :- p(b), not w(b)
      w(b) :- p(b), not m(b)

      Factual,
      minimal model: {p(a), w(a), m(b)}

   4. mother(a,b).
      father(c,b)
      sister(a,c).
      sister(X,Y) :- sister(Y,X).
      aunt(X,Y) :- sister(X,Z), parent(Z,Y).
      parent(X,Y) :- mother(X,Y).
      parent(X,Y) :- father(X,Y).

      Definite,
      minimal model:
      {mother(a,b),
      father(c,b),
      sister(a,c),
      sister(c,a),
      parent(c,b),
      parent(a,b),
      aunt(a,b)}