

Advanced Data Modeling

Summer Semester 2008

- Exercises VII -

*To be handed in before **2008-06-15, 23:59** via e-mail to
sschenk@uni-koblenz.de, subject line: [ADM] ...*

- 1) Let A be any set and A^2 its power set. Find two different orders O_1 and O_2 , such that (A^2, O_1) and (A^2, O_2) are lattices.
- 2) Show that $(2^{\text{BP}}, \subseteq)$ is a complete lattice.
- 3) Show that the non negative integers ordered by divisibility form a complete lattice. What are its supremum and infimum? Let least common multiple and greatest common divisor be operations on this lattice. Characterize the prime numbers using this lattice.
- 4) Are set union and intersection monotonic and/or continuous operations on the powerset of any set A ?
- 5) Hilbert's Paradox of the Grand Hotel
The Grand Hotel has infinitely many rooms and it is fully booked. However, n additional guests arrive. Show by induction that all n new guests will find a room in the hotel.

Now assume that infinitely many coaches, each with infinitely many seats arrive. All coaches are full of passengers. Show by transfinite induction that also all these new guests will find a room. Make sure that your proof also works for infinitely many trains with infinitely many cars with infinitely many seats etc.