Exercise
Data Science

Bayesian Statistics

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Bayes' rule 1

Sum rule: $p(C) = \int_M p(C, M) \, dM$

Product rule: $p(C, M) = p(M \mid C) \, p(C)$

$$\overbrace{p(M \mid C)}^{posterior} = \frac{\overbrace{p(C \mid M)}^{likelihood} \; \overbrace{p(M)}^{prior}}{\underbrace{p(C)}_{marginal\, likelihood}}$$

The sum rule and the product rule are the fundamental rules of probability. How can we derive from them Bayes' rule?

Bayes' rule 2

Let's assume a simple world whose inhabitants have one of two mindsets: With a probability of 0.6 they have a 'liberal' mindset and with a probability of 0.4 they have a 'conservative' one. The liberal minded, in this world, drink latte with a probability of 0.7 and mocha with a probability of 0.3 , and the conservatives drink mocha with a probability of 0.8 and latte with a probability of 0.2 .

We observe someone is drinking mocha. Can we use Bayes' rule to compute the probability of here/him having a liberal mindset?

Posterior distribution (LDA)

Generative Storyline:

$$\phi_k \sim Dir(\beta) \quad \forall k \in [1, K]$$

$$\theta_d \sim Dir(\alpha) \quad \forall d \in [1, D]$$

$$z_{di} \sim Cat(\theta_d) \quad \forall i \in [1, n_d],\ d \in [1, D]$$

$$w_{di} \sim Cat(\phi_{z_{di}}) \quad \forall i \in [1, n_d],\ d \in [1, D]$$

How would an appropriate inference algorithm look in pseudo code?

$$z_{di} \propto (n_{dk}^{\neg di} + \alpha) \frac{n_{kw_{di}}^{\neg di} + \beta}{n_{k.}^{\neg di} + V\beta}$$

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What are the parameters of the posterior distribution?